Advanced Modeling & Simulations Seminar Series
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The Nonlinear Harmonic Method

Efficient simulation of complex multi-row interactions in turbomachinery CFD

Stéphane VILMIN, PhD, Software Architect

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Overview

- Introduction
  Combining the basic periodicities to get the complex unsteady perturbations
- The mathematical tool:
  frequency-domain **nonlinear harmonic method** (NLH)
- Towards a **general** and **flexible** method
  - Pylon-Contrarotating rotors (CROR)
  - **Multi-rank NLH** for the interaction between adjacent and non-adjacent rows
  - Flexibility: **variable number of harmonics**
  - **Other sources** of disturbances (non induced by relative motion)
  - **Follow a flow disturbance across an arbitrary number of rows:**
    illustration: **from 0 harmonic to super-flexible harmonic simulation**

Introduction: time and space periodicities in turbomachinery

RELATIVE MOTION of cyclic symmetries implies **periodic space and time** variations.

A motion can also be **a change of position of a stationary row** (clocking).

Number of blades = base **CYCLIC SYMMETRY** of the rotating row

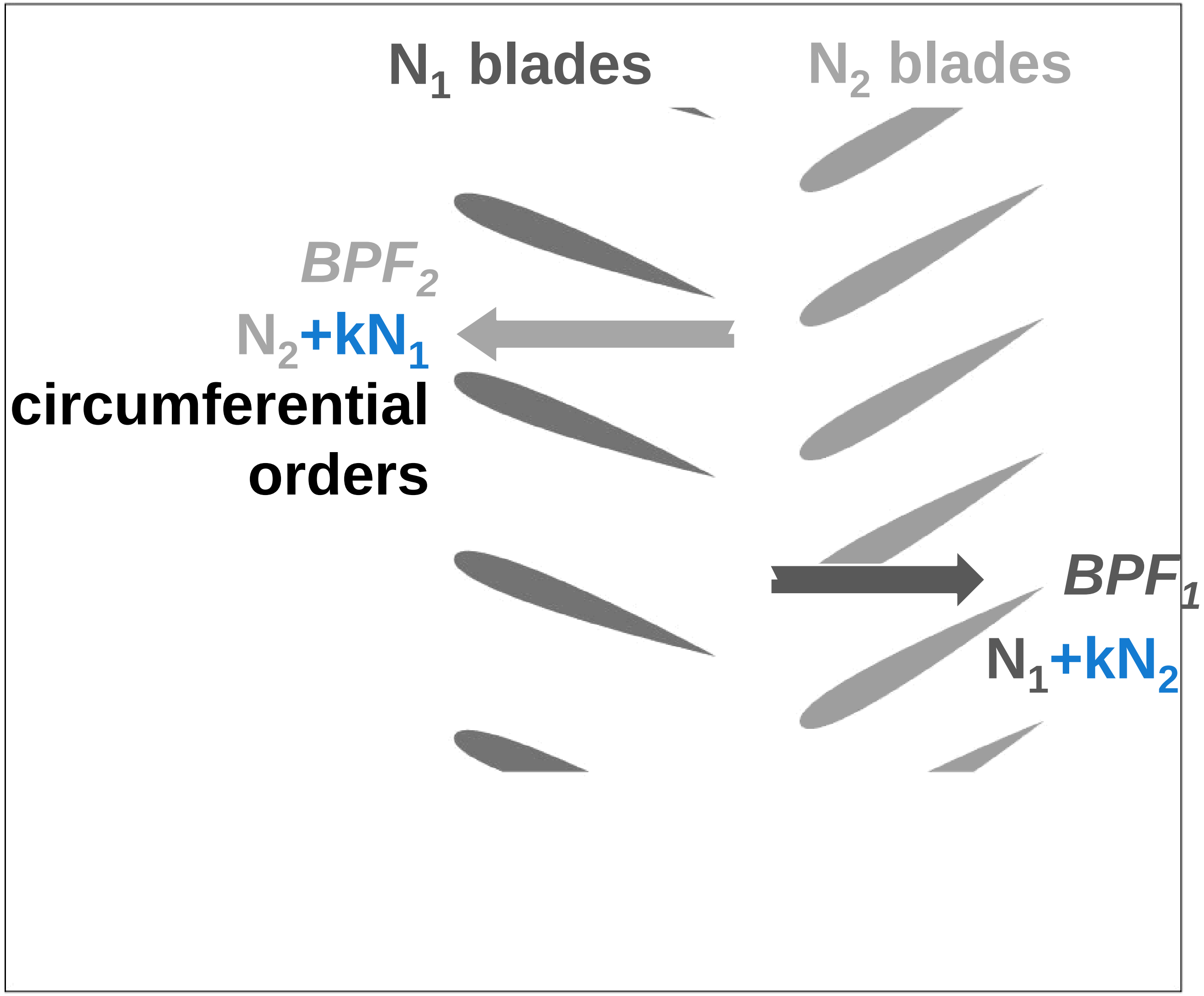
× ROTATION SPEED

➔ FREQUENCY **BPF** for periodic disturbance provoked by this row into the exterior world.
TIME perturbation

Difference of the cyclic symmetries

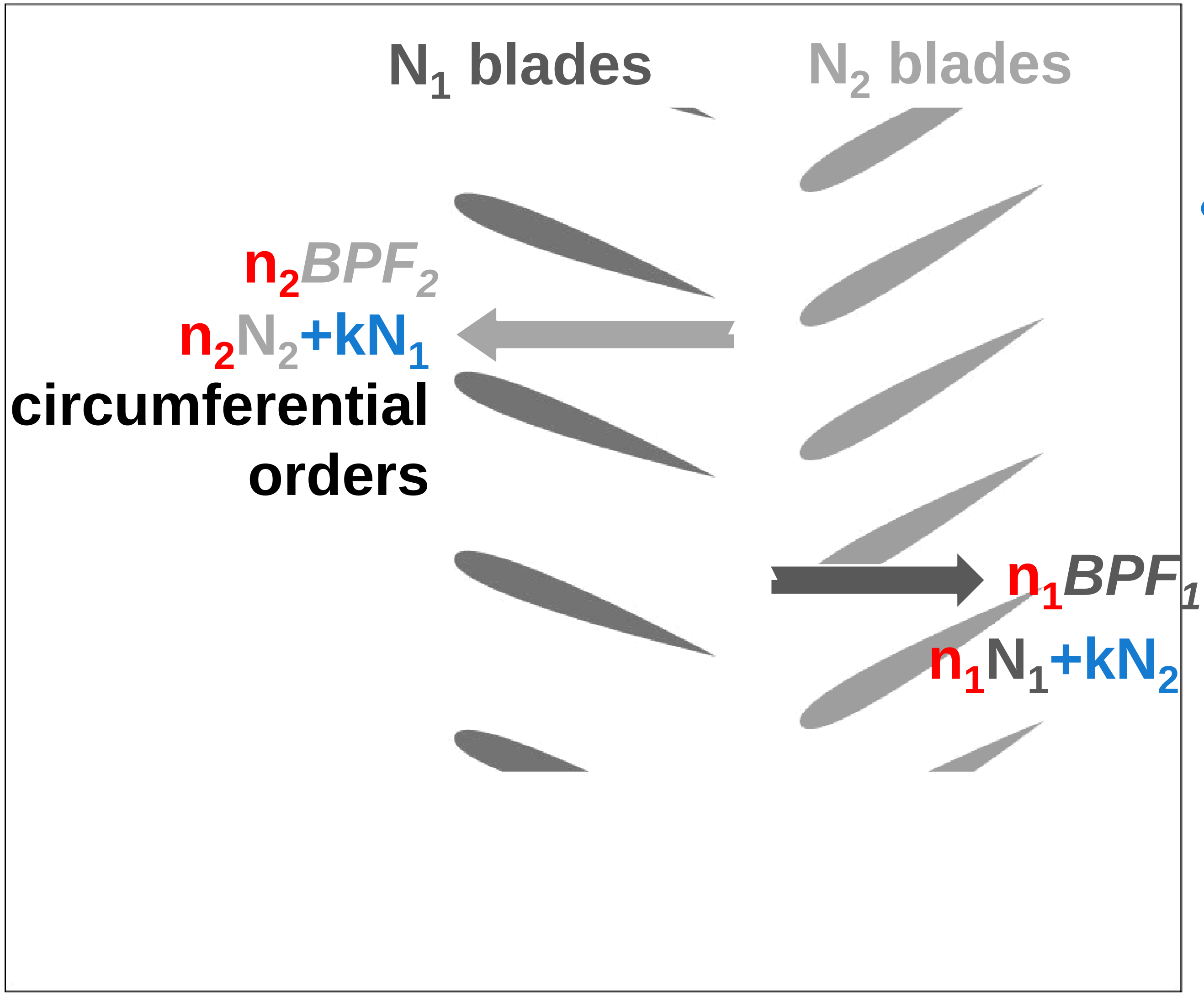
➔ **Modification** of the **CYCLIC SYMMETRY** of the unsteady flow in each row.
Still Cyclic but **periodicity in space at an instant** is no longer based on the number of blades of the current row.
SPACE perturbation, e.g. phase shift.

Combination of the periodicities in space



- The signal *in time* of frequency BPF is cyclically symmetric *in space and can be Fourier decomposed in space.*
- It includes **Additional combinatory** circumferential orders by interaction with the row where the BPF takes place

Combination of the periodicities in space



- The signal by definition is **periodic in time** and can be decomposed in n **time FOURIER components**.

Effect of the combination

4-blade ROTOR **3-blade STATOR**

$BPF_1 = 4 \times$Speed(Rotor)

$N_1 + kN_2 = 4 + k \times 3$ circumferential orders

For k=-1, ω /circ.order = ω / *1* = *4* ×Speed(Rotor)
For k= 0, ω /circ.order = ω / *4* = *1* ×Speed(Rotor)
For k= 1, ω /circ.order = ω / *7* = *0.57* ×Speed(Rotor)

For same BPF_1

A ***m*-cyclic** pattern rotating at rotor-speed would induce a m×BPF.
It induces a BPF, so it rotates at BPF/m.

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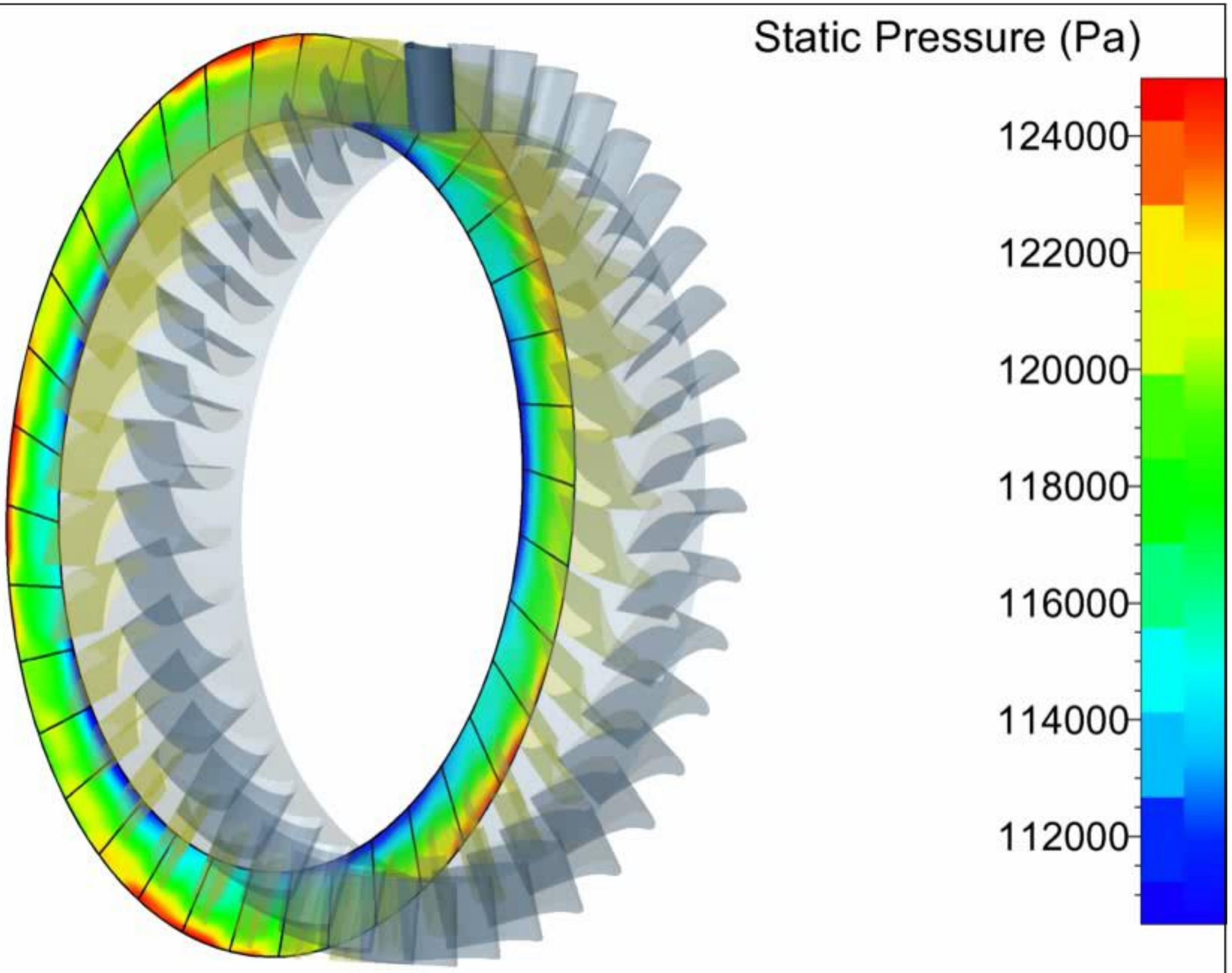
36-blade rotor
41-blade stator

In the stator,
a 36-41= 5-cycle
is visible,
rotating at
36 / 5
= 7.2
$RotSpeed_{rotor}$



Methodology: the baseline Nonlinear Harmonic Method (NLH).

Baseline NLH (Vilmin et al., ASME Turbo 2006) = base by Li He (1998)

= Solving the Unsteady Reynolds-Averaged Navier-Stokes by casting the transport equations into the frequency domain.

The flow variables $U=\left(r, r\vec{v}, rE\right)$ are split into a **time-mean + periodic fluctuations**, associated with a frequency, e.g. BPF in turbomachinery. N harmonics are considered for each frequency.

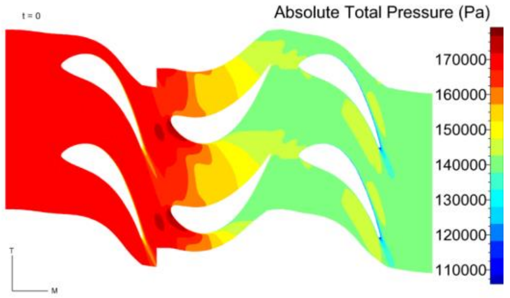
$$U\left(\vec{r},t\right)=\overline{U}\left(\vec{r}\right)+\sum U'\left(\vec{r},t\right)$$

$$U'\left(\vec{r},t\right)=\sum_{k=1}^{N}\left(\tilde{U}_k\left(\vec{r}\right)e^{i\omega_k t}+\tilde{U}_{-k}\left(\vec{r}\right)e^{-i\omega_k t}\right)$$

- - - The flow variables solved by NLH

Flow Fourier decomposition

$$U(\vec{r},t) = \overline{U}(\vec{r},t) + \sum \sum_{k=1}^{N} \left[\tilde{U}_k(\vec{r}) e^{j\omega_k t} + \tilde{U}_{-k}(\vec{r}) e^{-j\omega_k t} \right]$$



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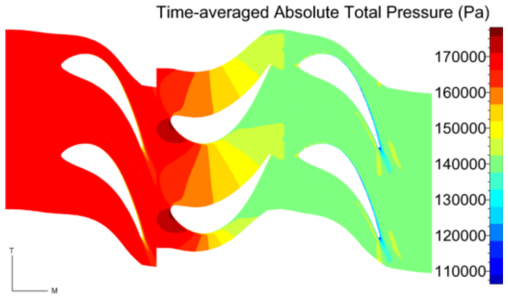

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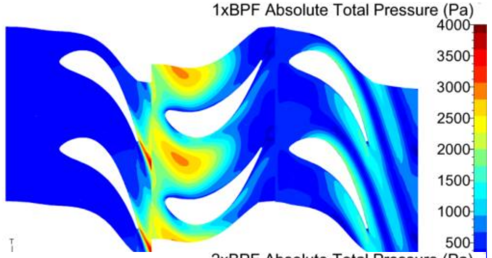
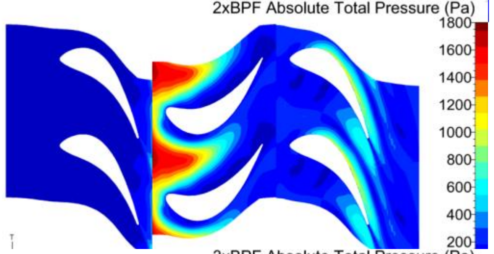
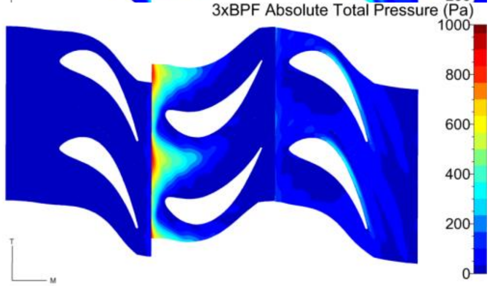




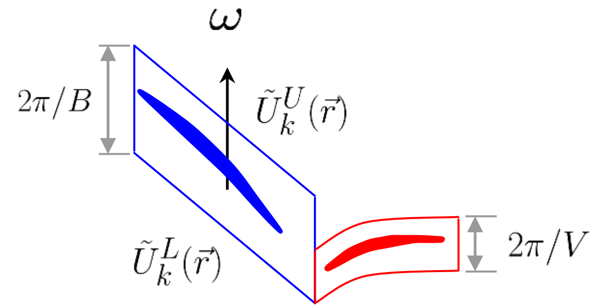


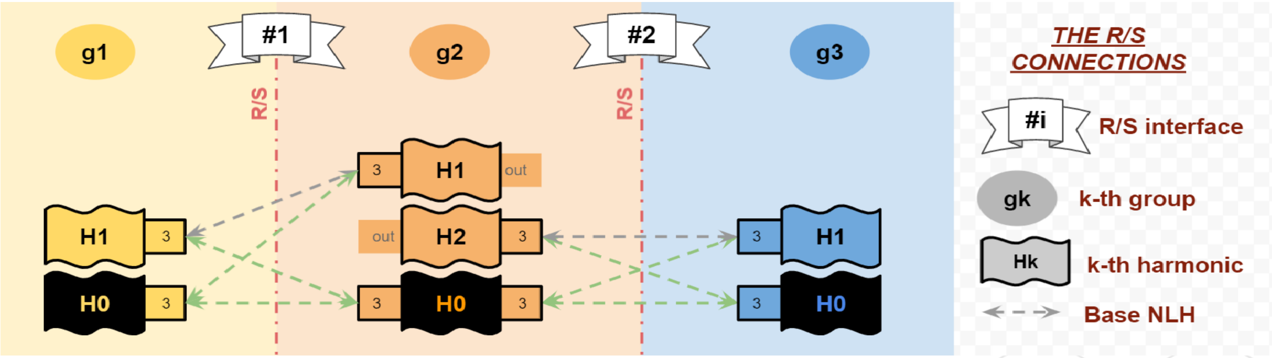
Reconstruction in time of the absolute total pressure field at an axial turbine's 50% span with NLH rank-2 computation (3 harmonics per perturbation)

Only one blade channel is calculated per row

- Periodic: the **phase-lag** boundary condition is trivial in the frequency domain, using the *interblade phase angle*, σ.
- The flow in the whole row can therefore be **easily reconstructed from the one-channel solution**, by application of the phase lag of each harmonic.
- **Rotor/stator interface** is a cross-connection of the time-mean and harmonics, and applies a non-reflective treatment.

$$\tilde{U}_k^U(\vec{r}) = \tilde{U}_k^L(\vec{r})e^{jk\sigma}$$
$$\sigma = -2\pi V/B$$





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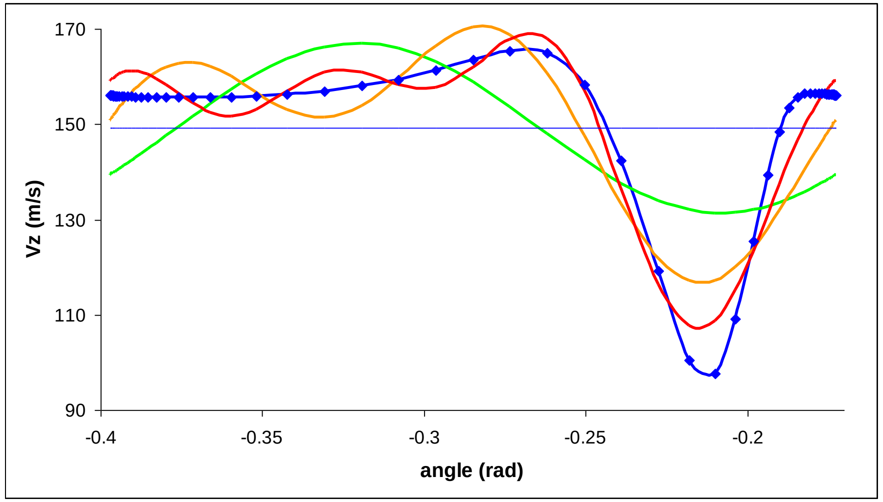
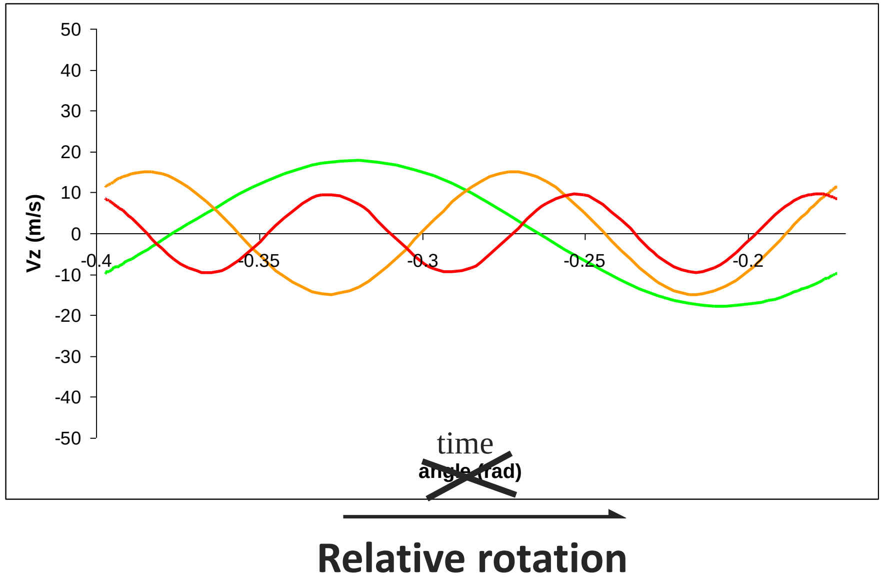
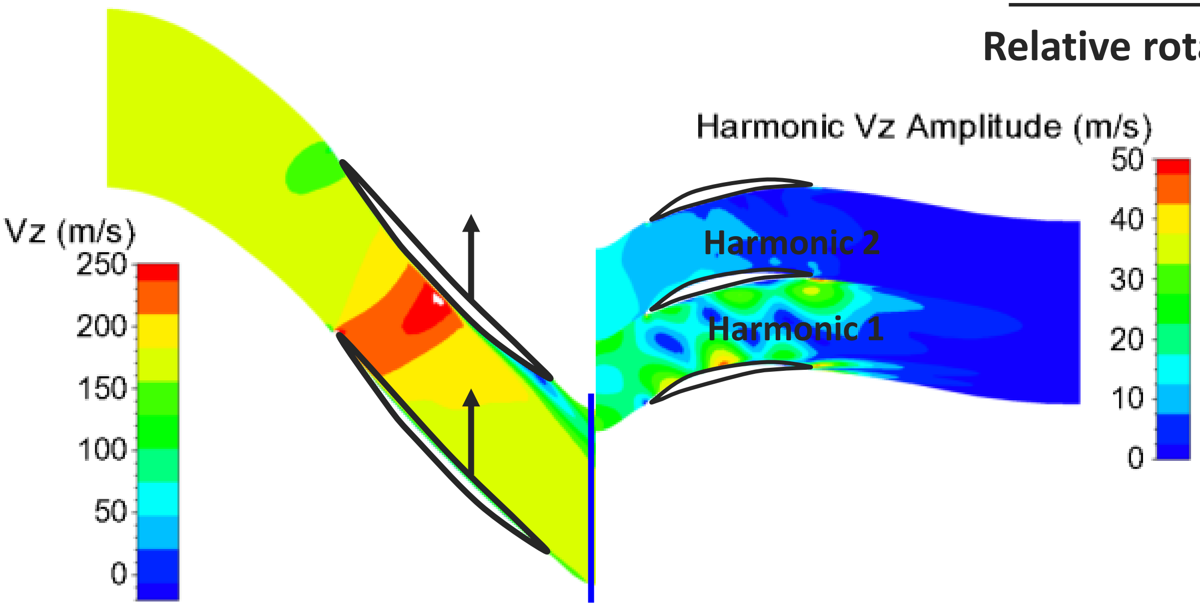
Rotor/Stator interface treatment: applies Space Fourier







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Mathematical modeling

- Replacing the variables in the NS equations by their decomposition :

$$\frac{\partial U}{\partial t} + \frac{1}{Vol}\sum F_C \vec{S} + \frac{1}{Vol}\sum F_V \vec{S} + Q = 0$$

Time-mean flow by time averaging:

$$\sum \overline{F_C \vec{S}} + \sum \overline{F_V \vec{S}} + Vol\overline{Q} = 0$$

$$\overline{F}_C \vec{S} = \begin{pmatrix} (\overline{\rho \vec{v}} \cdot \vec{S}) \\ (\overline{\rho \vec{v}} \cdot \vec{S})\,\overline{v}_x + \overline{p} S_x \\ (\overline{\rho \vec{v}} \cdot \vec{S})\,\overline{v}_y + \overline{p} S_y \\ (\overline{\rho \vec{v}} \cdot \vec{S})\,\overline{v}_z + \overline{p} S_z \\ (\overline{\rho \vec{v}} \cdot \vec{S})\overline{H} \end{pmatrix} + \mathbf{Det_c}$$

Harmonic equation by linearization:

$$i\omega\tilde{U}Vol + \sum \tilde{F}_C \vec{S} + \sum \tilde{F}_V \vec{S} + Vol\tilde{Q} = 0$$

$$\widetilde{F}_C \vec{S} = \begin{pmatrix} \widetilde{\rho \vec{v}} \cdot \vec{S} \\ \tilde{v}_x \left(\overline{\rho \vec{v}}\cdot\vec{S}\right) + \overline{v}_x \left(\widetilde{\rho \vec{v}}\cdot\vec{S}\right) + \tilde{p} S_x \\ \tilde{v}_y \left(\overline{\rho \vec{v}}\cdot\vec{S}\right) + \overline{v}_y \left(\widetilde{\rho \vec{v}}\cdot\vec{S}\right) + \tilde{p} S_y \\ \tilde{v}_z \left(\overline{\rho \vec{v}}\cdot\vec{S}\right) + \overline{v}_z \left(\widetilde{\rho \vec{v}}\cdot\vec{S}\right) + \tilde{p} S_z \\ \tilde{H} \left(\overline{\rho \vec{v}}\cdot\vec{S}\right) + \tilde{H} \left(\widetilde{\rho \vec{v}}\cdot\vec{S}\right) \end{pmatrix}$$

$$\overline{f'g'} = 2\sum_i \left(\mathrm{Re}\left(\tilde{f}_i\right)\mathrm{Re}\left(\tilde{g}_i\right) + \mathrm{Im}\left(\tilde{f}_i\right)\mathrm{Im}\left(\tilde{g}_i\right)\right)$$ Li He (1998)

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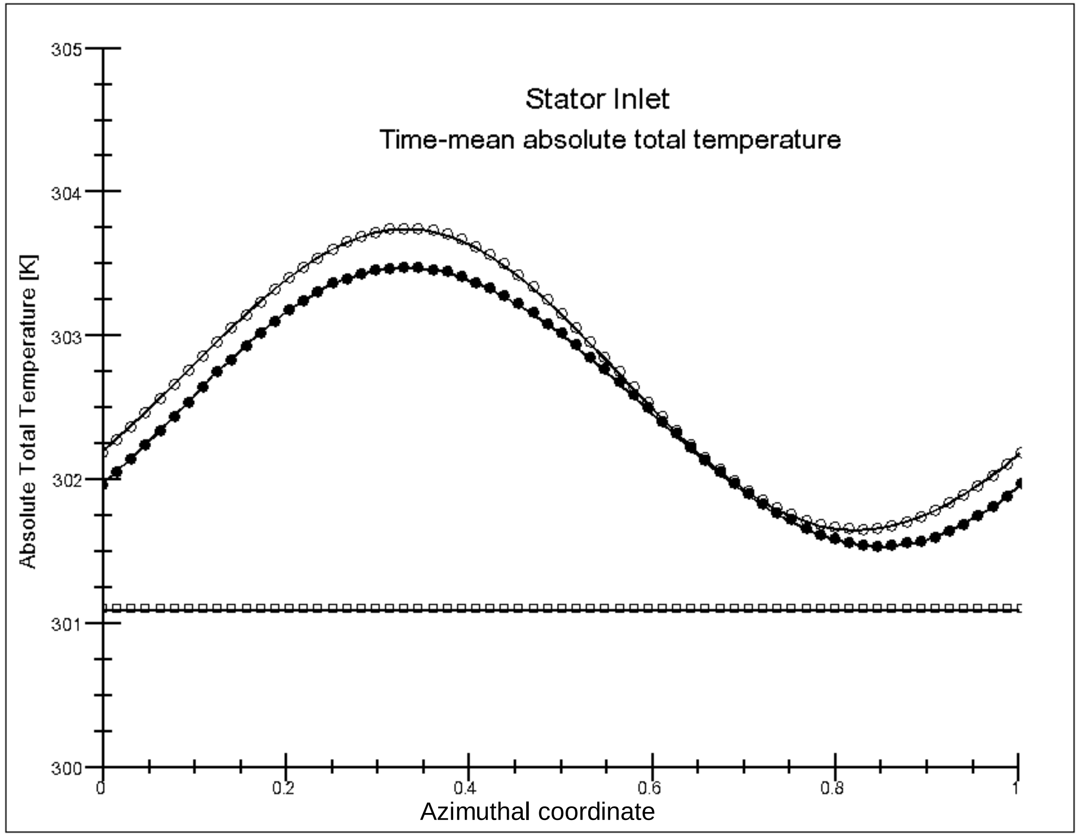
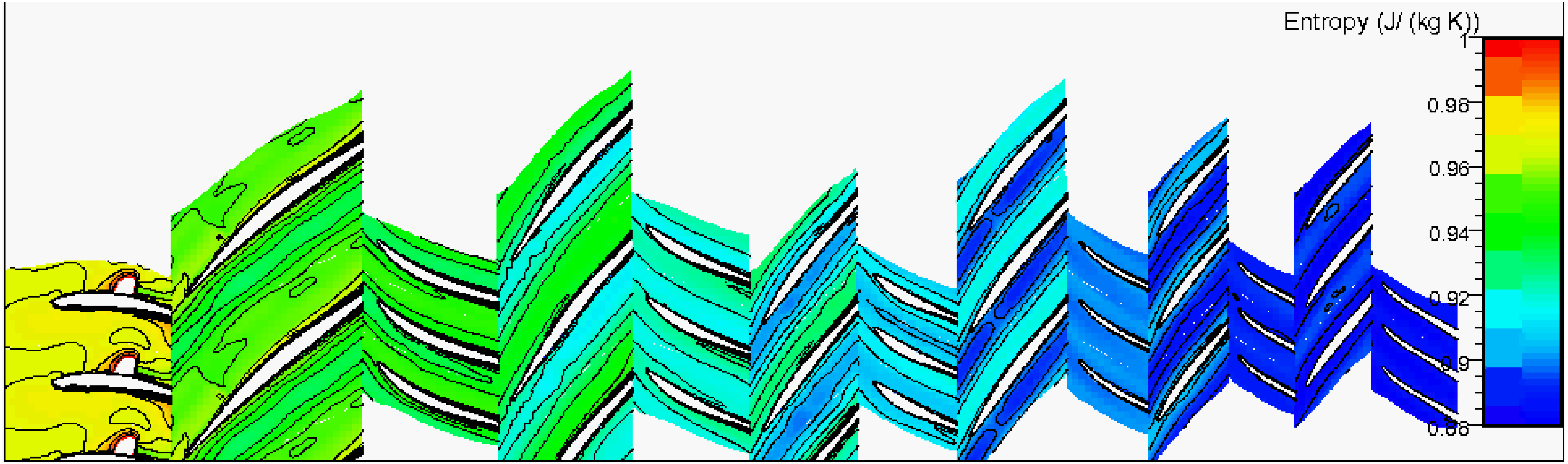
Influence of the unsteadiness on the time-averaged flow



- ○ Harmonic method
- ● Nonlinear full unsteady method
- □ Mixing-plane method

Clocking

- Based on Li He’s method (2002): zero-frequency perturbation ➔ clocking harmonics
- Impact on row position **on the time-mean flow** and direct post-processing of the unsteady flow.



Modifying relative position of a rotor impacts on time-mean solution inside adjacent rotors (same for stator). Time reconstruction can be performed for a selected position.

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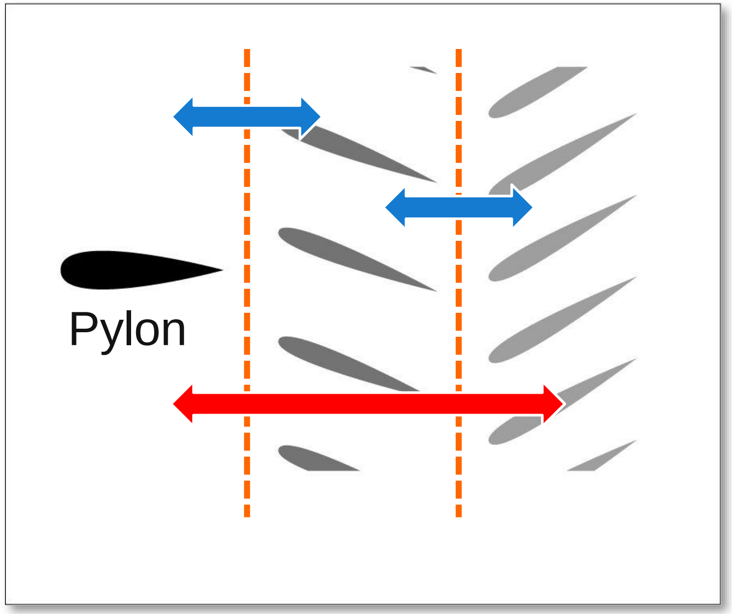
Motivation for a **generalization** of the NLH: more perturbations

Extension to “multi-rank” periodic effects, and not only between co-stationary rows.

CROR



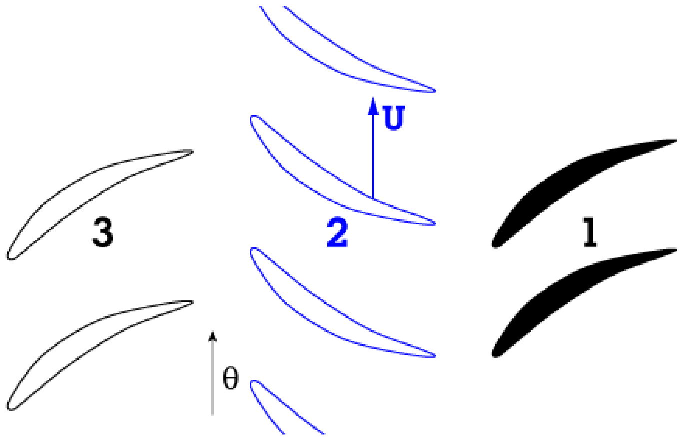
Rank 1 = effects due to adjacent
OK for baseline NLH

Rank 2 = effects due to adjacent
of adjacent.

In the 2nd rotor, the effects from the relative rotation of the pylon exist and the associated time-harmonics can be solved and vice-versa.

Extensions to multi-rank periodic effects

MORE time frequencies, which are also more complete:



In row$_2$

All the linear combinations of BPF1 and BPF3 are taken into account and can be solved.

In row$_1$ and row$_2$ and row$_3$

The linear combinations of the numbers of blades of row$_1$ and row$_3$ give new spatial frequencies associated with a time frequency
= new time harmonics with same frequency but different interblade phase angle, that include the clocking effects.

The generalized R/S connection: multi-rank NLH

For $Row_i/Row_{i+1}/Row_{i+2}$, simulates effects between Row_p and Row_q $p,q \in \{i,i+1,i+2\}$

Connection of the flow
between the two sides R and L of the R/S interface

$$\tilde{U}_{j,\mathrm{R}} = \sum_{k=-N}^{N} \left(\int \tilde{U}_{f(k,j),\mathrm{L}} \, e^{-In_{\theta,k,j}\theta_{\mathrm{L}}} d\theta \right) e^{In_{\theta,k,j}\theta_{\mathrm{R}}}$$

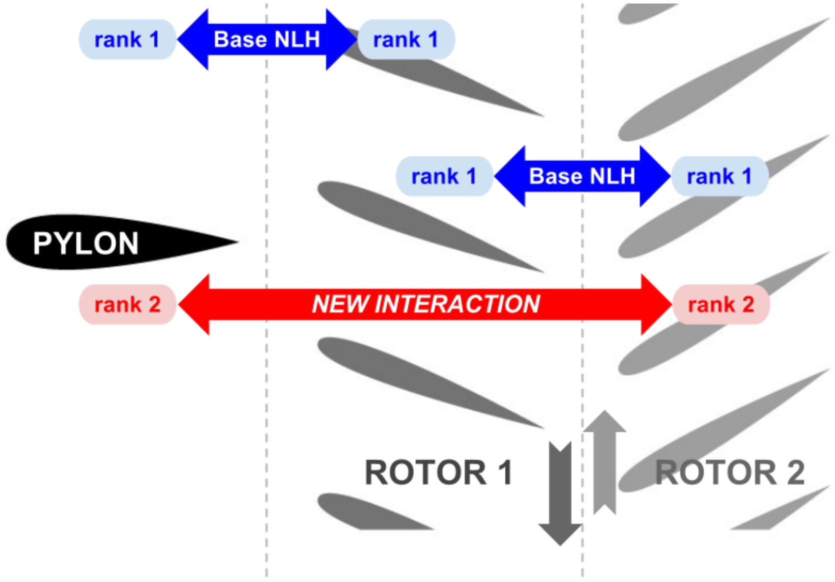
j and *f* = time indices (including the time mean, index 0)

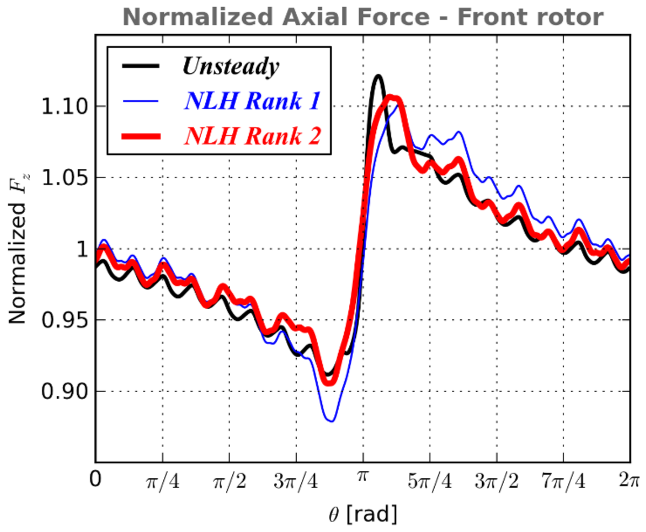
N = number of harmonics

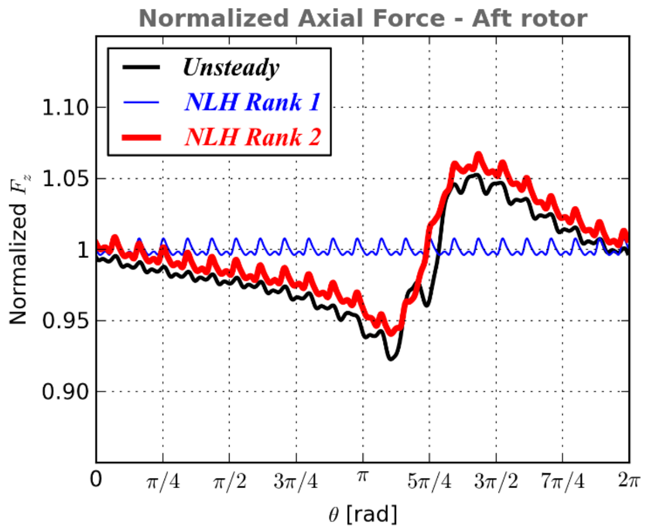
$n_{\theta,k,j}$ = azimuthal (circ.) number or spatial mode

See Vilmin et al., 2013, ASME TurboExpo paper and International Journal of CFD.

Application for a CROR + the mounting pylon







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NLH FLEXIBILITY: a property of the generalized R/S connection

1 - Give the **NUMBER OF HARMONICS PER RANK:**
This will process the **number maxi of harmonics per disturber**.

2 - Toggle on/off **INTERACTION** and **order of interaction** between the harmonics
= **max value** that **j** and **k can have** for harmonic jG_m+kG_n

3 - Toggle on/off **limitation** of **stator/stator effects** to the **time-mean flow only**.

4 – Manual de-/activation of harmonics before launching the simulation

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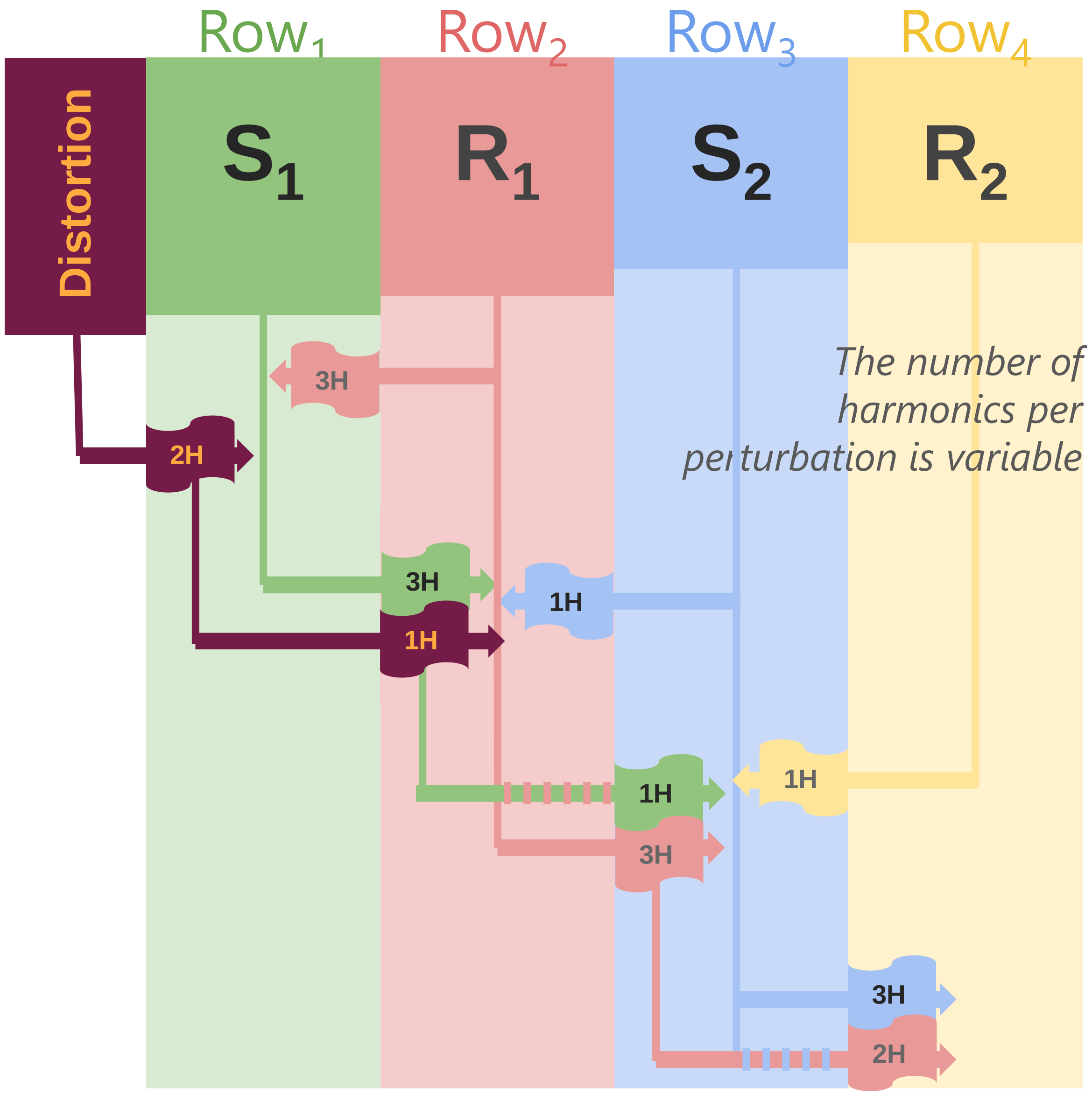
Set the level of approximation of the perturbations



The HARMONIC METHOD approximation:

- Extent of the disturbance = the **rank.** Maximal rank = 2 rows.
- Accuracy = the **number of HARMONICS** that are used for the Fourier decomposition of the periodic perturbations.

The accuracy is specified by the user and can vary,
i.e. = function(perturbation), and/or = function(rank).

No unsteady impact if 0 harmonics for the perturbation.

Here, **R_2** effects into **S_2** less accurate than effects from **S_2** into **R_2**.

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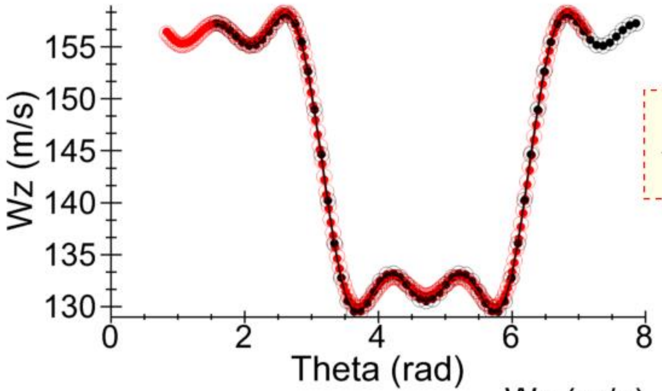
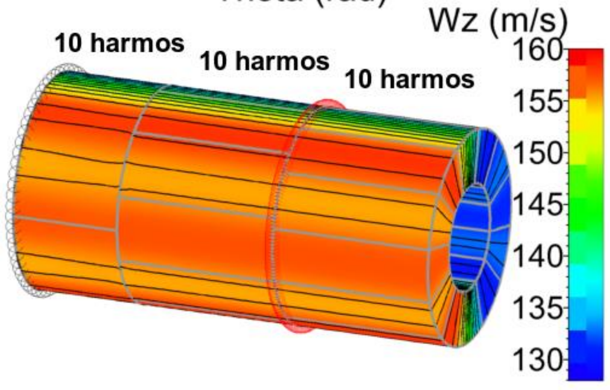
Flexibility: activate / deactivate harmonics

Some cases do not require a whole set of harmonics.

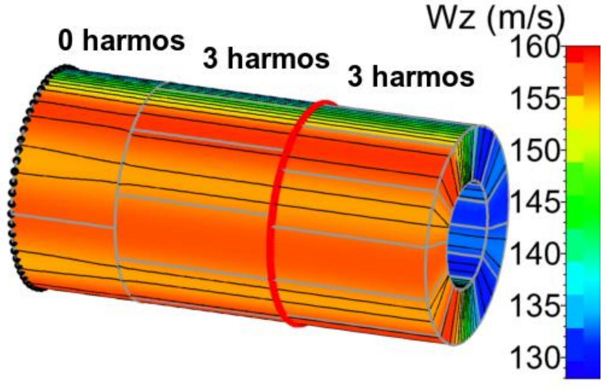
- In this 1.5 stage:
  Same solution is achieved by limiting the number of harmonics to the significant harmonic contents (periodicities **1, 3 and 5 are kept**, while disgarding the 2 and 4), and deactivating the upstream running perturbations.
- See Mehdizadeh et al., inlet distortion in engine fan stage (ETC 2019)



$$P_0(\theta)_{\mathbf{inlet}} = P_0 + \alpha\left(sin\theta + \frac{1}{3}sin3\theta + \frac{1}{5}sin5\theta\right)$$



Full set



Flexible set

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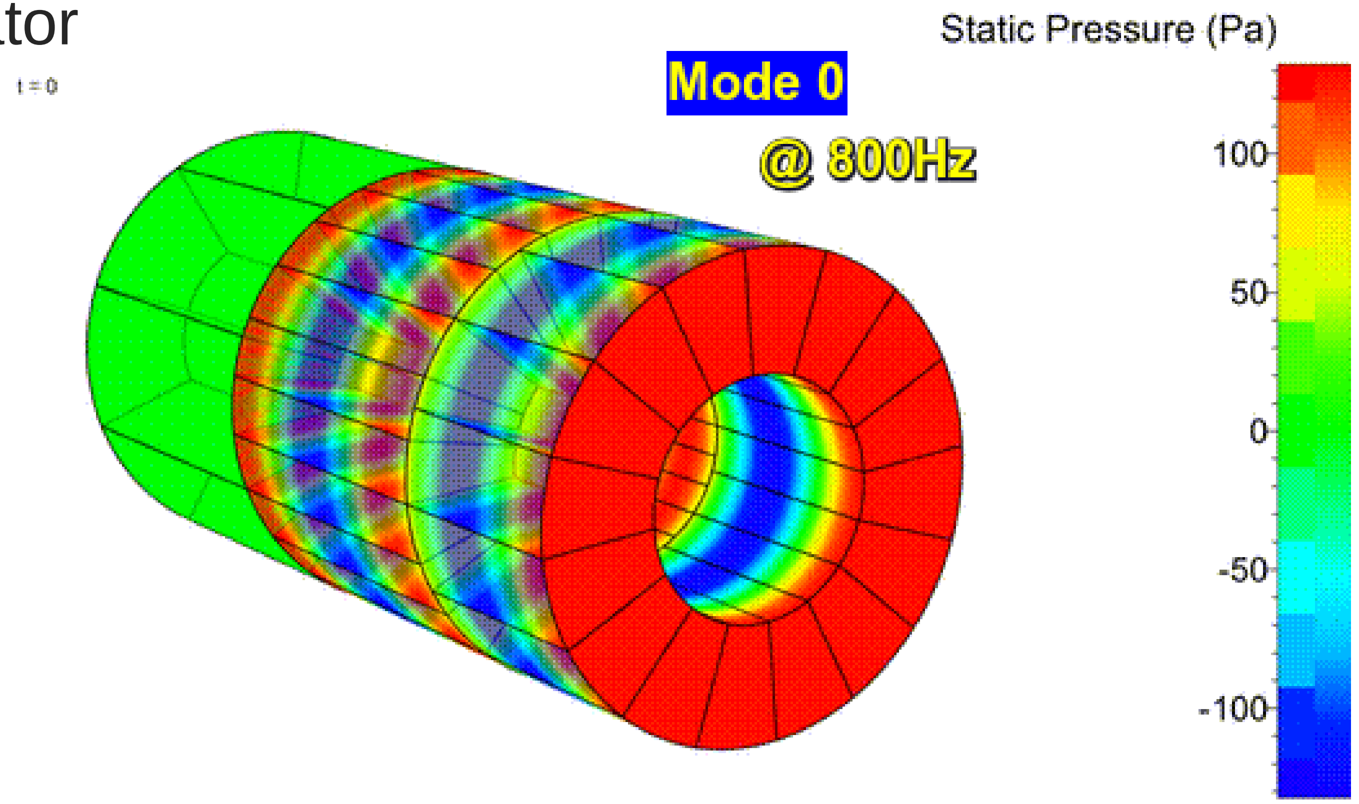
Injection of flow distortion at the inlet of Stator-Rotor-Stator

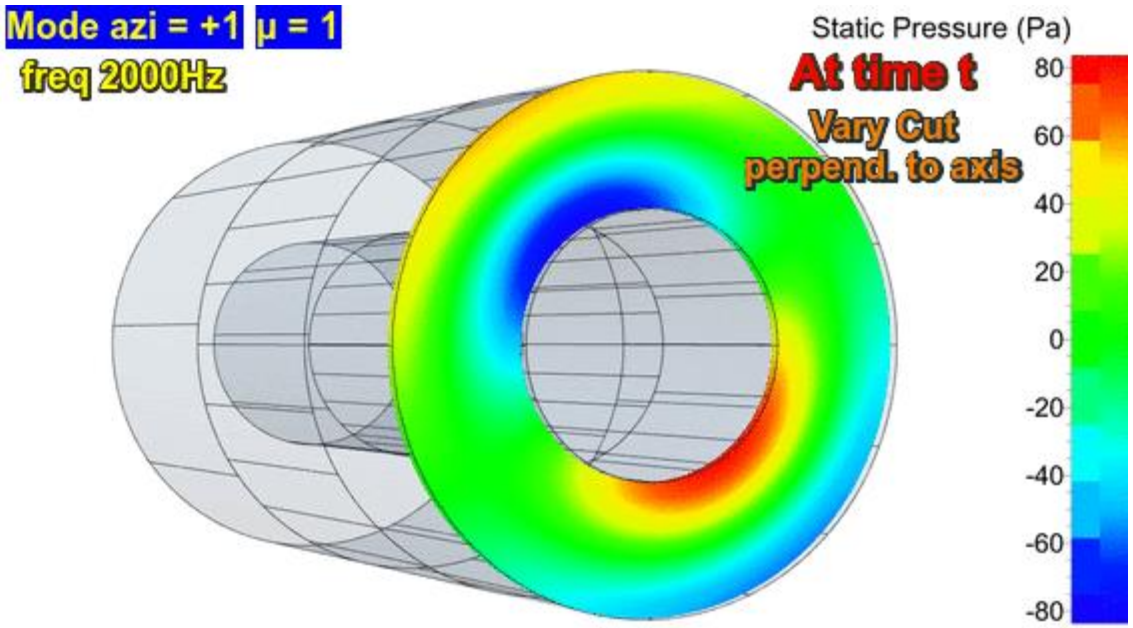
Linearized boundary conditions + non-reflecting treatment.

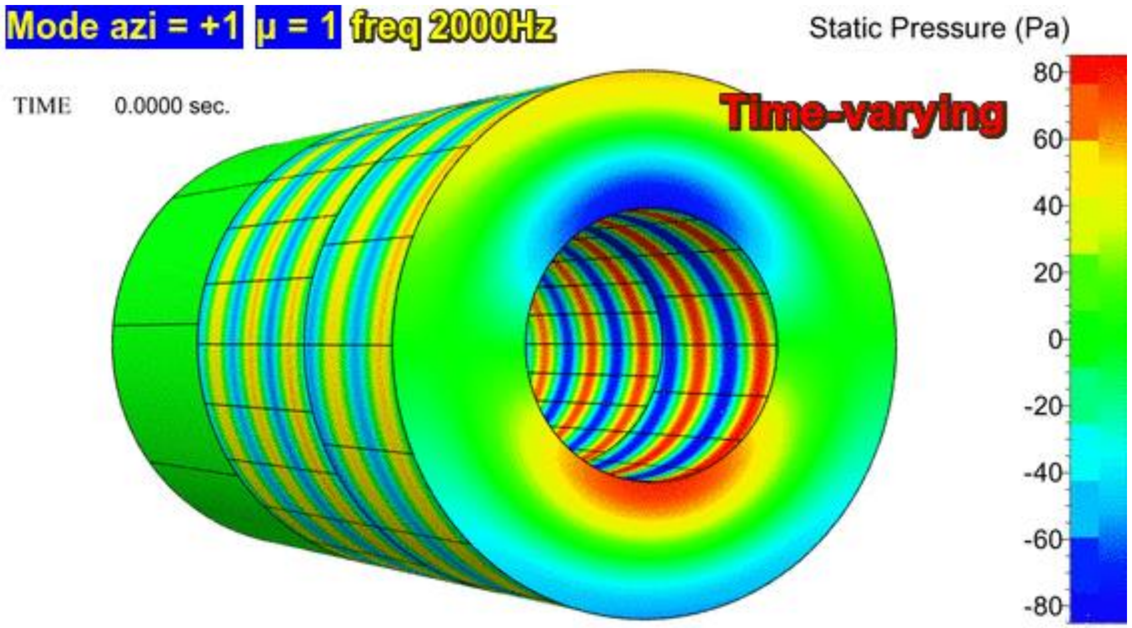
- Example: plane acoustic wave transported by constant axial flow across a stator/rotor/stator



Injection of flow distortion at the inlet of Stator-Rotor-Stator

- Tyler-Sofrin mode ((“Axial flow compressor noise studies”,1962) transported by constant axial flow across stator/rotor/stator.





- See Mehdizadeh (ETC 2019) for inlet total pressure distortion across an engine fan stage

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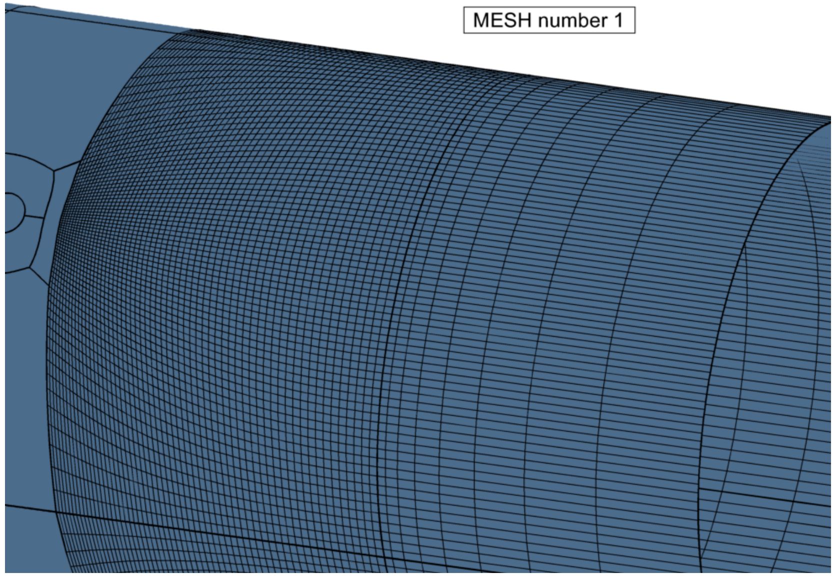
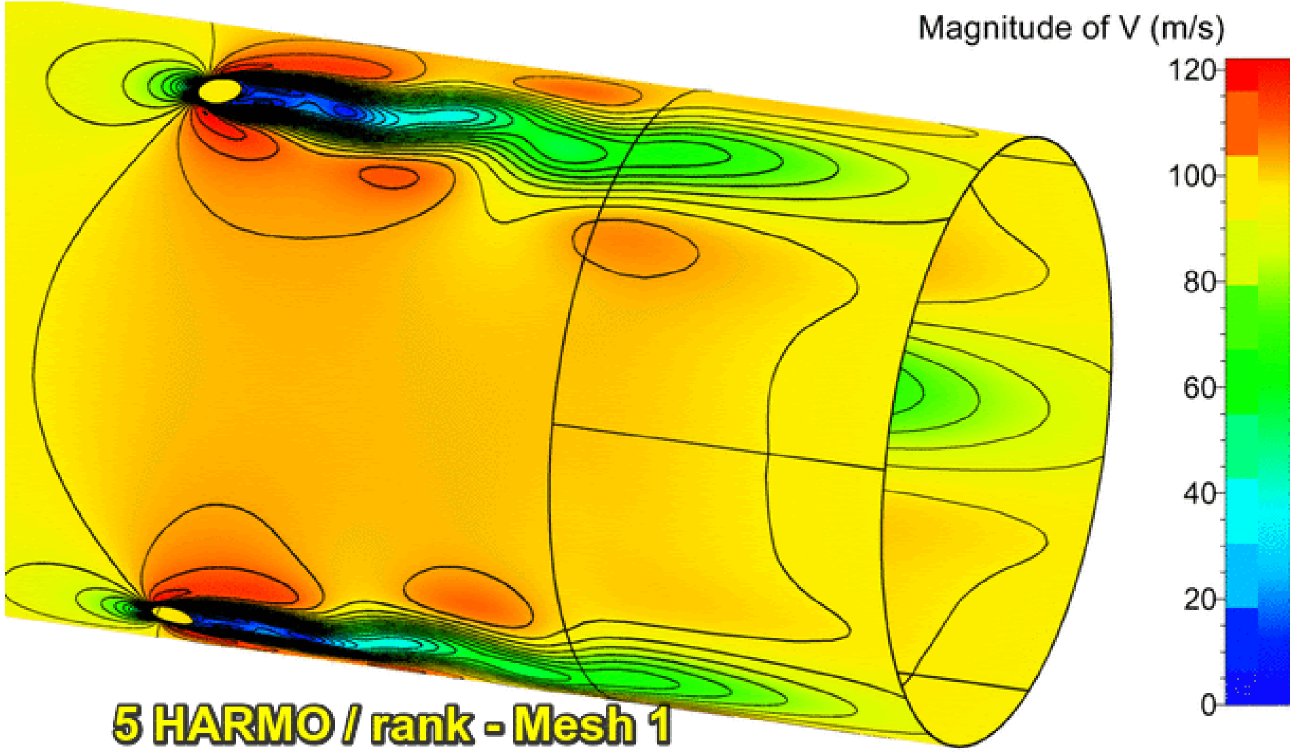
The NLH inner perturbation

1 - Inject harmonics from within a row

2 - The goal is to reproduce dynamically generated, i.e. “self-excited”, time perturbations, such as a Von Karman vortex street or a rotating stall, by specifying the frequency in the absolute domain and an associated periodicity along the azimuth, which might be zero.

Non-BPF Von Karman perturbation interacting with BPF





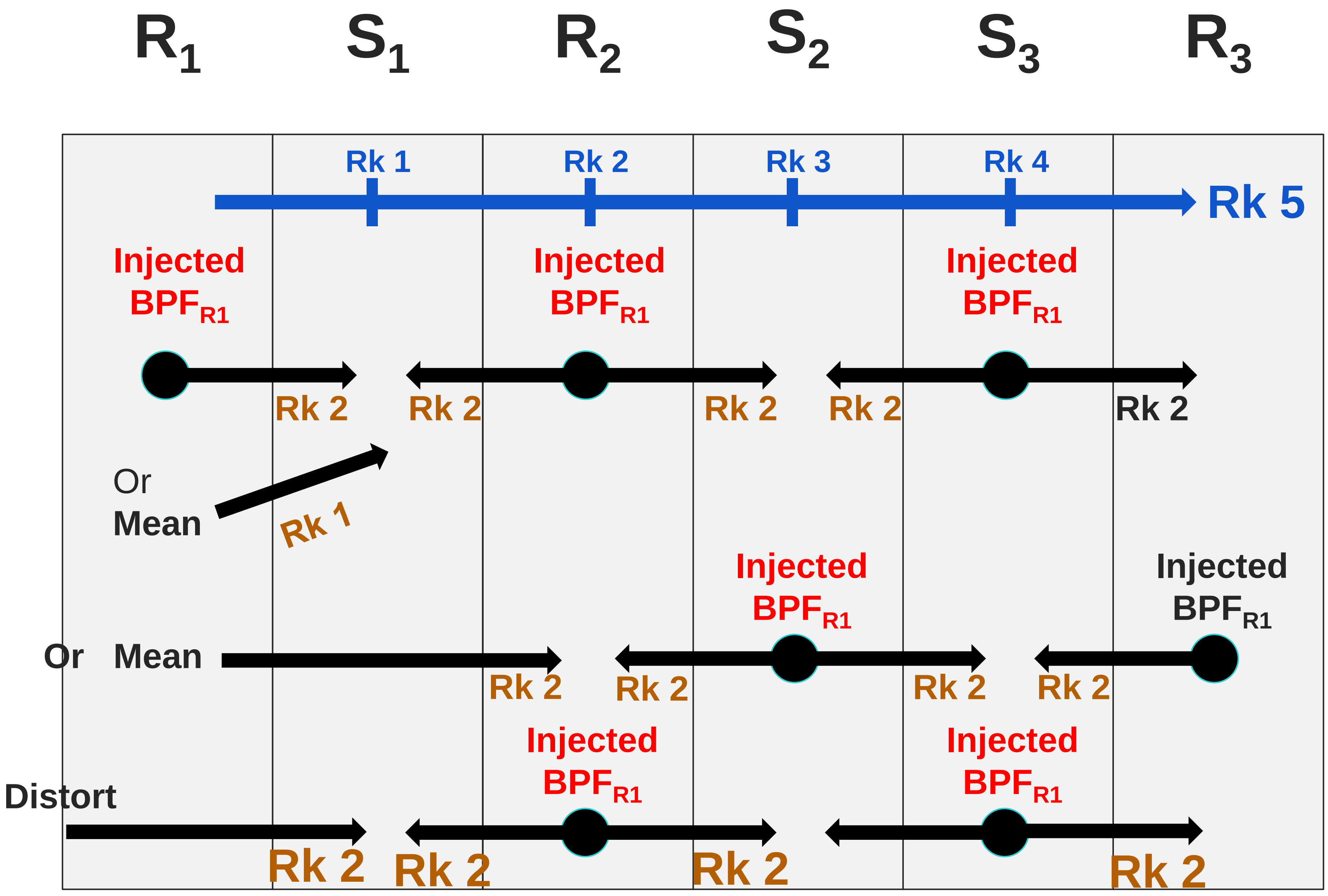
Like in slide 27, the **non-BPF signal (here of periodicity 0) interacts with the BPF** that exists in each row, otherwise the signal would not be fully connected at the R/S interface

The NLH inner perturbation pushed further

3 - Rank-2 + flexibility can be used to get a **broader picture of multirow flows,** without modifying the R/S mechanisms.

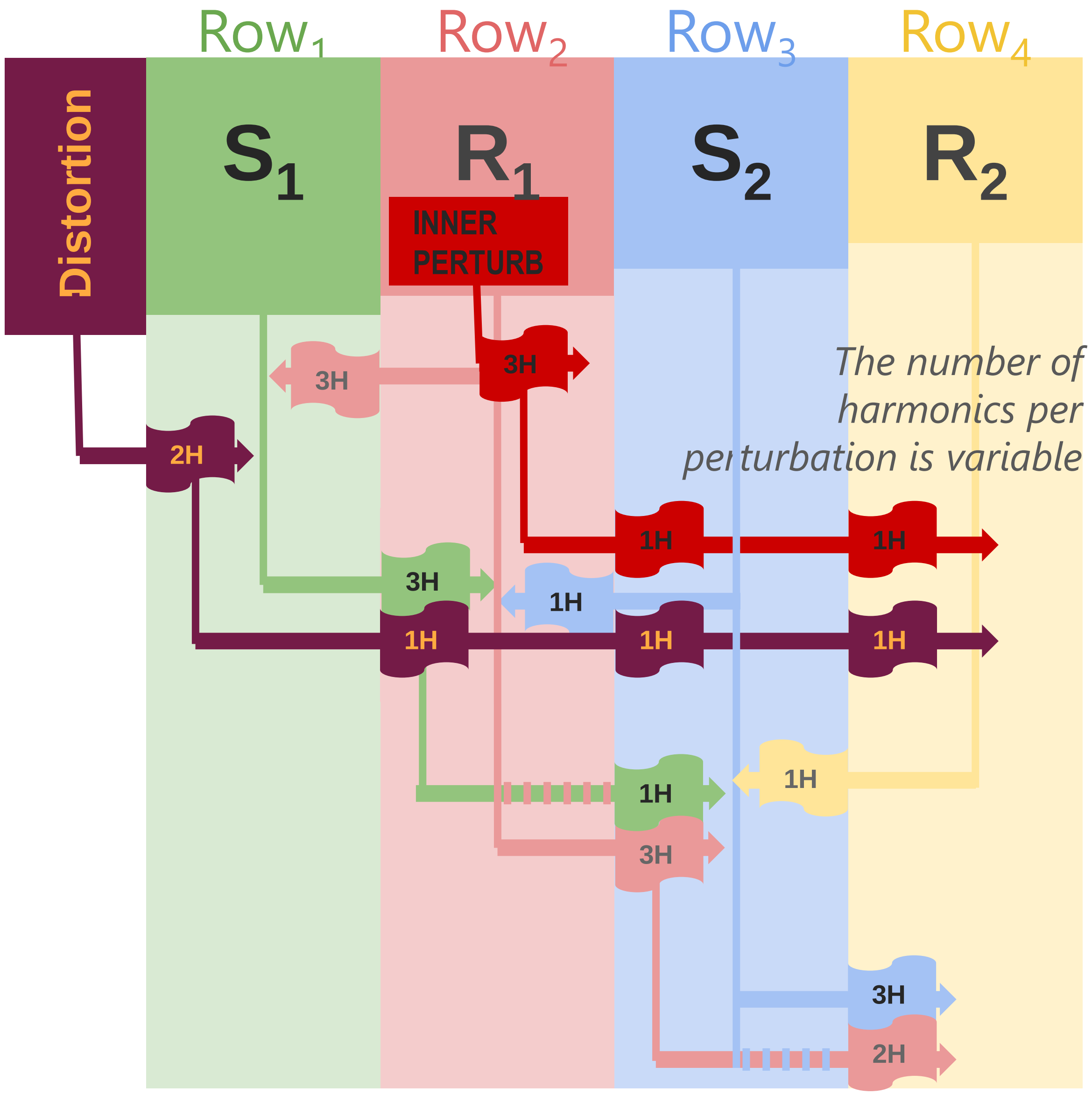
BPF signal = an inner perturbation ➔ can be injected.

Just needs to find the boundary values at the R/S (connection)
(otherwise the signal at the injected frequency would not be fed and would be null)



Merging of the identical harmonics (same frequencies and same interblade phase angles) is applied.

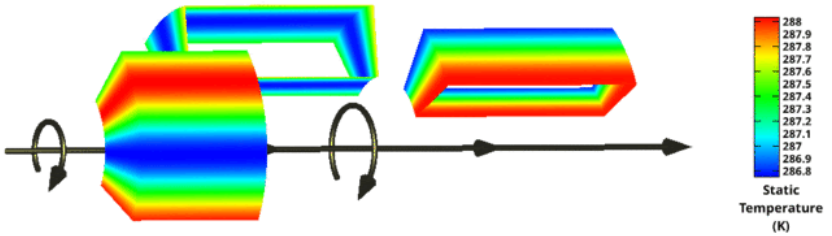
A broader picture for different perturbations in a multi-row



Extent of the disturbance = the **rank**.

Maximal rank = as high as needed

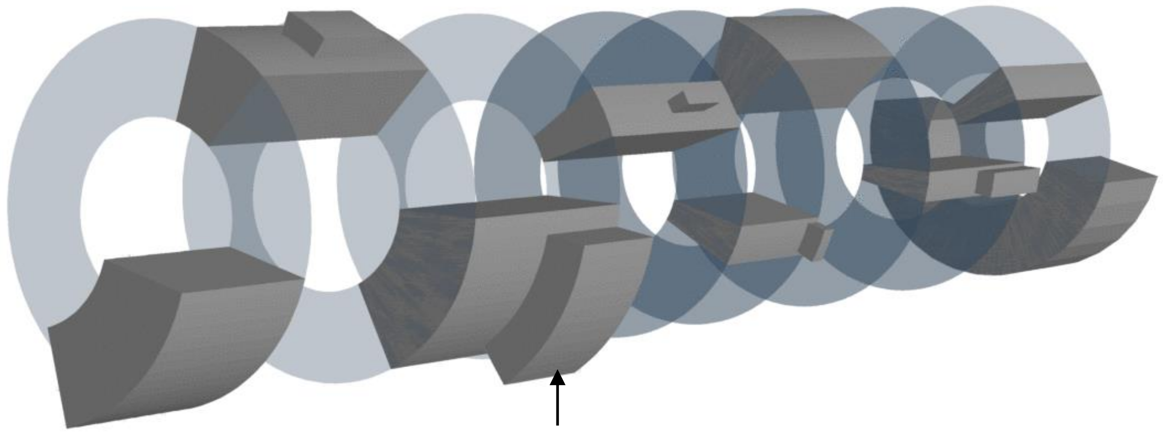
Example:



Inlet **6-periodic** distortion transported by a constant axial flow across rows of periodicities **5-10-12**

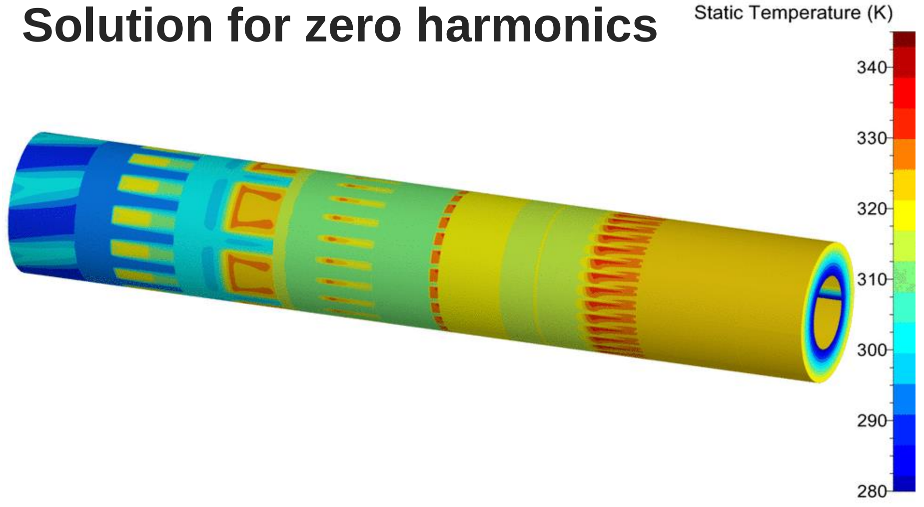
Broader perturbation fields: a simple example

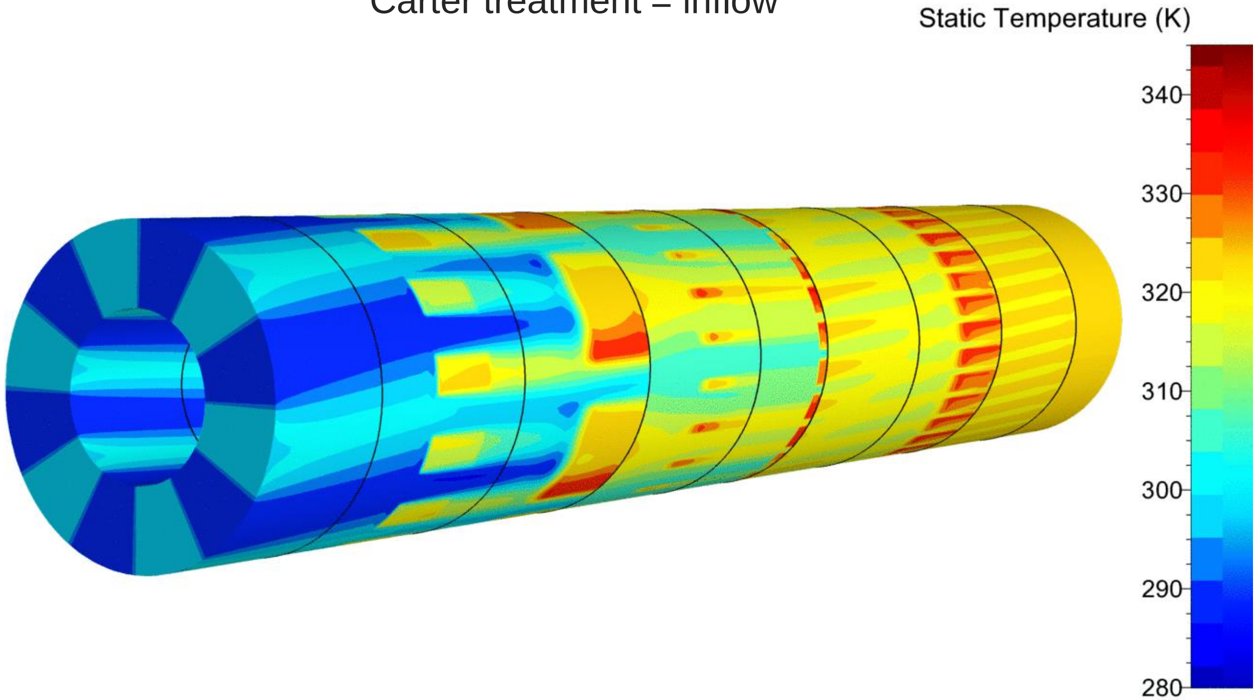
R_1 — S_1 — S_{CC} — S_{HPT} — R_{HPT} — S_{1LPT} — R_{LPT} — S_{2LPT} — $S_{exhaust}$



Carter treatment = inflow

Solution for zero harmonics

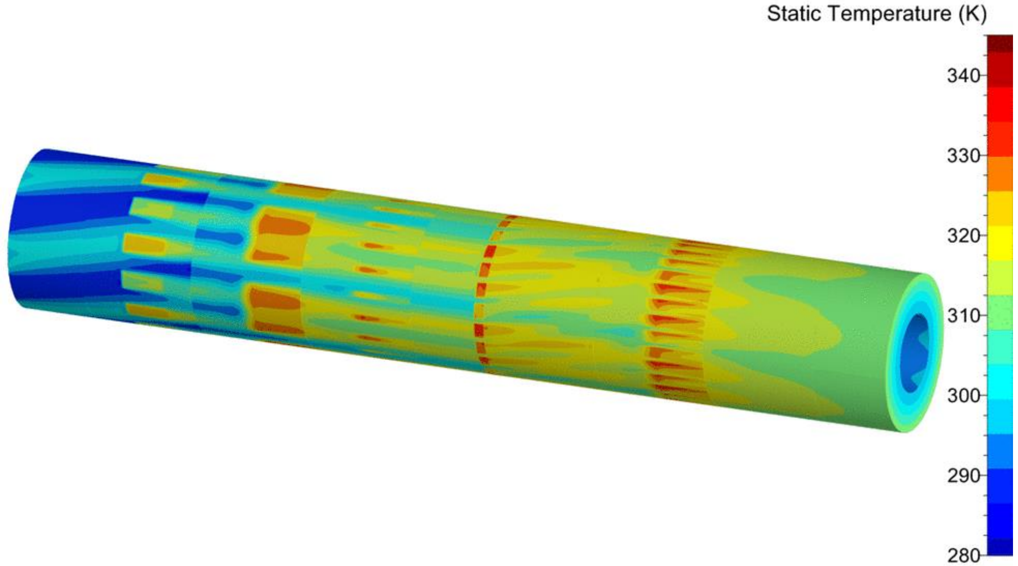


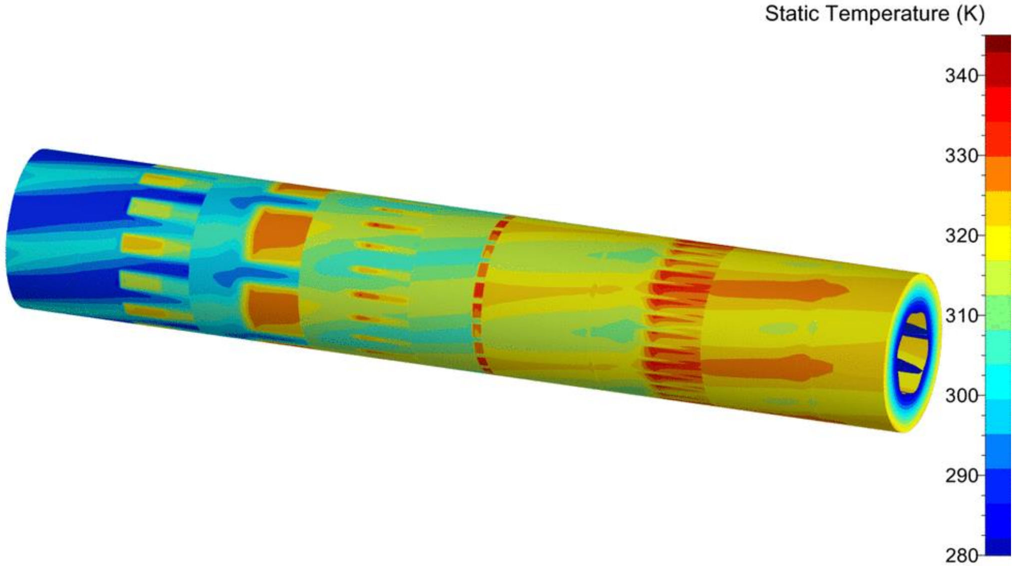


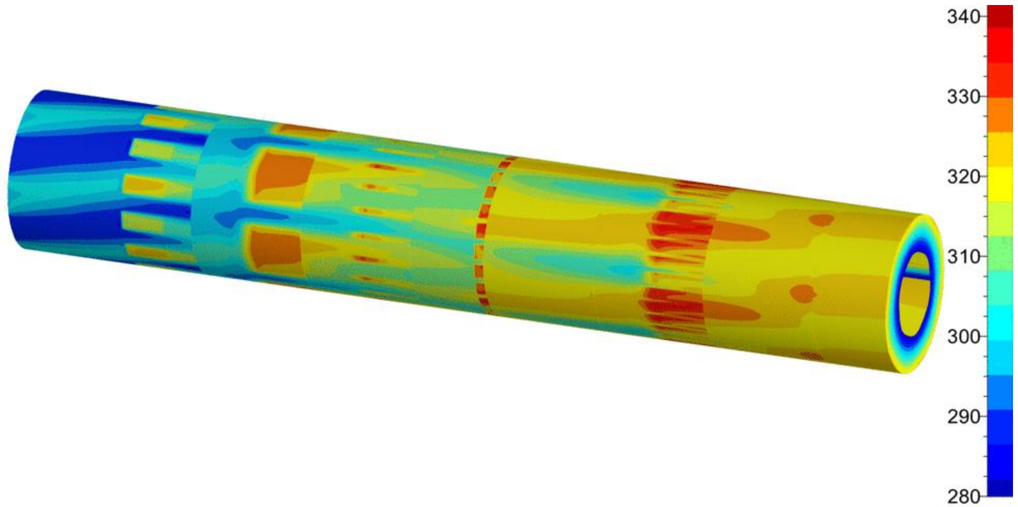
Solution for rank maxi = 2

Harmonics go into CC, from R_1 and S_1. In S_{HPT}, effects are visible from CC and S_1. R_{HPT} is impacted by CC and S_{HPT}. There is a visible interaction mode in the first S_{LPT}. The periodicity in azimuth is the difference 24-20 = 4. The speed of rotation is the BPF of R_{HPT} divided by 24-20, ie. 6 times the rotation speed of the rotor.

Solutions for rank maxi value = as you wish







The transported wake from the first row, and the injection flow at S_{CC}, can be modelled as far as desired and monitored in the exit row.

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